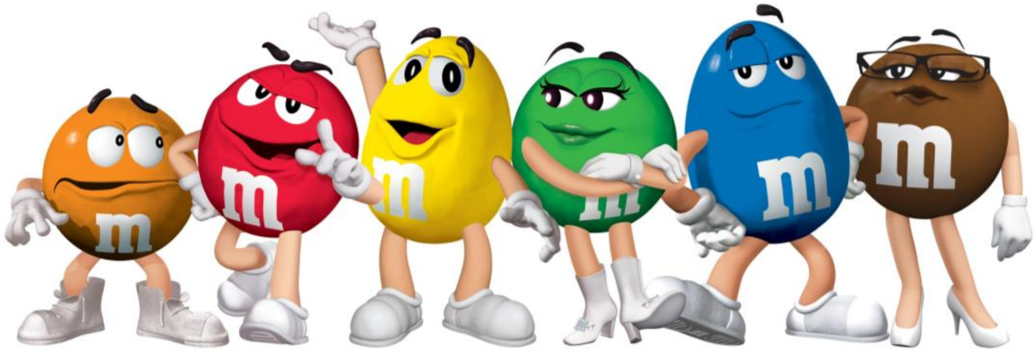
Name ______________________________ Date ___________
AP Biology Mr. Collea

M&M’s Statistics



Part I. Am I getting what I pay for?

Do M&M bags labeled 3.14oz (89 g) actually contain that weight? If not, what is a typical weight and how much variability is there from bag to bag? To answer those questions, we will collect data on some bags of M&M’s. Each group will weigh each bag and record the weight in the table below. Your task is to use the tables below to calculate and interpret the **mean** and **standard deviation** for the weights of bags labeled **3.14 oz / 89.0g**. Use your standard deviation calculation to complete the graph on the next page.

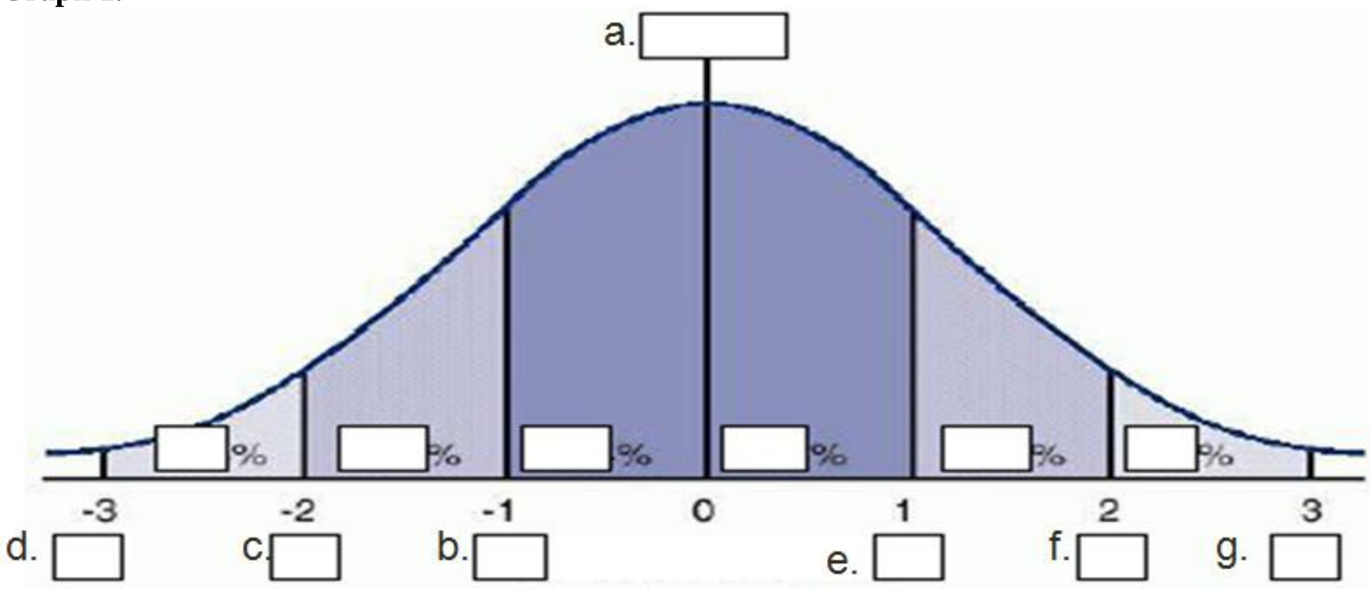
Table 1.

Bag	Weight (x)	Mean ($\bar{x}$)	$x-\bar{x}$	$(x-\bar{x})^2$
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
			$\Sigma(x-\bar{x})^2$ →	

Formula: Standard Deviation

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

Graph 1.



So what does all of this mean?

For normally distributed data, ______ of data points lie between **±1** standard deviation of the mean, ______ of data points lie between **±2** standard deviations of the mean and ______ of data points lie between **±3** standard deviation of the mean.

OR

For normally distributed data:

_______ of the bags should weigh between ______ g and ______ g

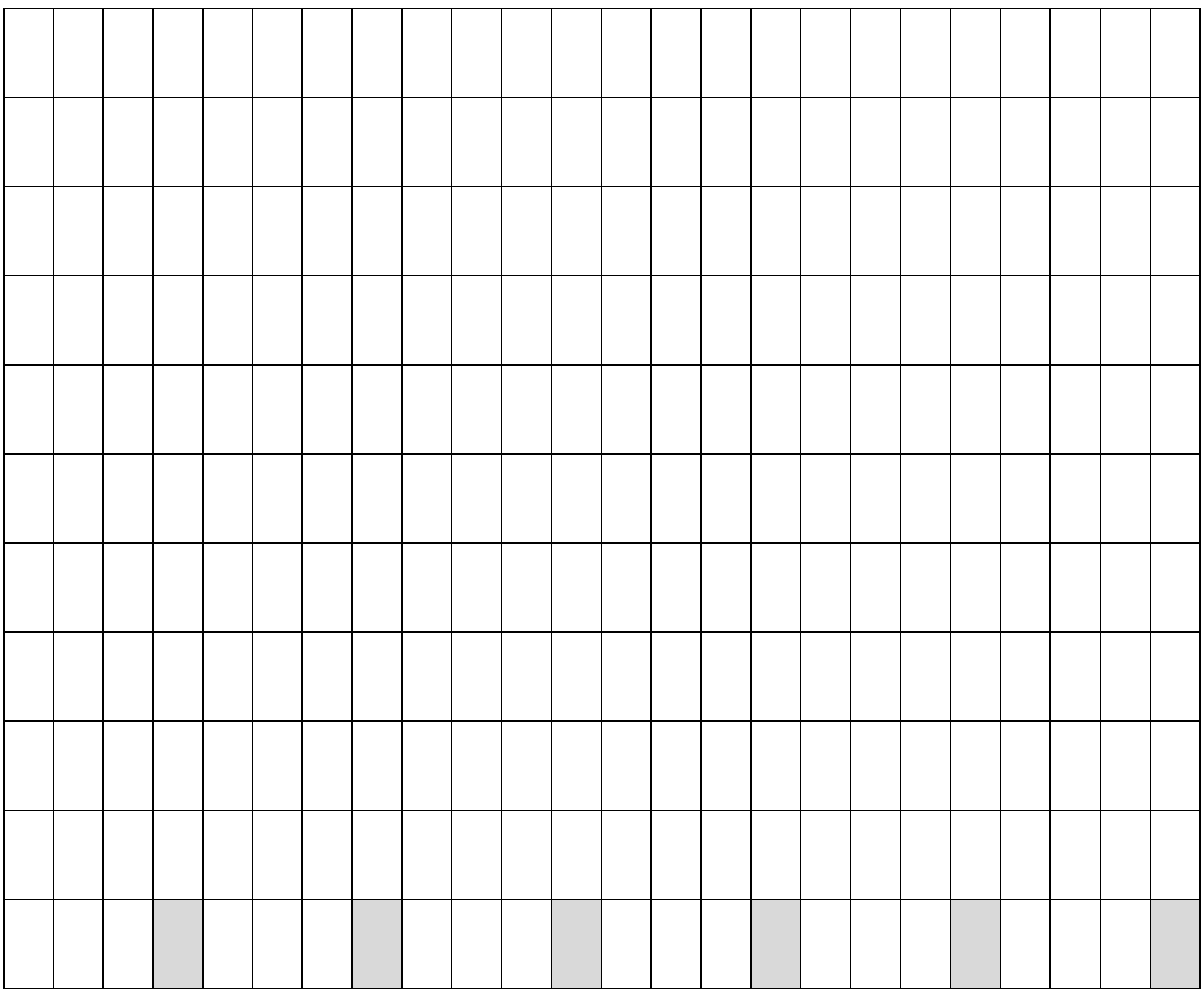
_______ of the bags should weigh between ______ g and ______ g

_______ of the bags should weigh between ______ g and ______ g

Part II. Distribution of Colors in Plain M&M's

Color	#	Total	%
Brown			
Blue			
Orange			
Green			
Red			
Yellow			

Graph 1. Distribution of Colors in Plain M&M's



Part III. Chi-Square Analysis of Colors in M&M's

<u>Background Information</u>:

Consider a trait that exhibits the pattern of simple dominance. If we were to cross two heterozygous individuals (Ex: Aa x Aa), then we would *expect* a 3:1 ratio of dominant to recessive phenotypes in the offspring. But what if we actually did this cross and did not get the expected 3:1 ratio? The difference from the *expected* ratio could be due to random chance or some type of sampling error. But it is also possible that the difference from the *expected* ratio is due to the fact that our original expectation was incorrect (i.e., *the trait does not actually exhibit the pattern of simple dominance*).

How can we determine which of these is the most likely cause of the difference between our expectation and our actual observation?

We can conduct a Chi-Square ($\chi 2$) analysis!

When starting a Chi-Square analysis, we must first identify the null hypothesis. A null hypothesis is a prediction that something is **<u>not</u>** present, that a treatment will have **<u>no effect</u>**, or that there is **no <u>SIGNIFICANT</u> difference** between what is ***expected*** and what is ***observed***. Another way of saying this is the hypothesis that an ***observed*** pattern of data and an ***expected*** pattern are effectively the same, differing only by chance, not because they are truly different.

The null hypothesis is for a Chi-Square analysis is **ALWAYS** the same:

There is <u>NO</u> significant difference between the observed and expected results.

OR

Any difference between the observed and expected data is due to CHANCE.

The goal of Chi-Square analysis is to **ACCEPT** or **REJECT** this <u>null hypothesis</u>.

Once we have calculated a value for the Chi-Square, we will compare it to a table of critical values. If the calculated Chi-Square value is ***smaller*** than the critical value, we **ACCEPT** our null hypothesis because our data is consistent with what we would expect - any slight difference is due to chance. If the calculated Chi- Square is ***larger*** than the critical value, we **REJECT** our null hypothesis because our data is too different from what was expected to explain the differences by chance - there must be some other explanation so guess what…you're about to learn something!

This investigation will let you practice using the Chi-Square test with a "population" of familiar objects, M&M® candies. Later on, we will use this same method to analyze the results of other experiment including genetic crosses.

Objectives:

After completing the investigation, you should be able to:

- *write and test a null hypothesis that pertains to this investigation.*
- *determine the degrees of freedom for an investigation(s).*
- *calculate the $\chi 2$ value from observed data and expected data.*
- *determine if the Chi-Square value exceeds the critical value and if the null hypothesis is accepted or rejected.*
- *explain the importance of large sample sizes when conducting statistical analysis of any kind.*

Problem:

Have you ever wondered why the package of M&Ms you just bought never seems to have enough of your favorite color? Why do you always seem to get the package of mostly brown M&Ms? What's going on at the Mars Company? Is the number of the different colors of M&Ms in a package really different from one package to the next? Or, does the Mars Company do something to insure that each package gets the correct number of each color?

Here is some information from the M&M website:

Table 1: Expected % of Color in M&Ms

% Color	Milk Chocolate
Brown	**13%**
Blue	**24%**
Orange	**20%**
Green	**16%**
Red	**13%**
Yellow	**14%**

Degrees of Freedom = ________________
(n – 1)

Table 2: Chi-Square Table

	Degrees of Freedom							
p	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0.05	**3.84**	**5.99**	**7.82**	**9.49**	**11.07**	**12.59**	**14.07**	**15.51**
0.01	6.64	9.32	11.34	13.28	15.09	16.81	18.48	20.09

One way that we could determine if the Mars Company is true to its word is to sample a package of M&Ms and do a type of statistical test known as a "goodness of fit" test. This type of statistical test allows us to determine if any differences between our observed measurements *(counts of colors from our M&M sample*) and our expected (*what the M&M website claims*) are simply due to chance or some other reason (i.e. *the Mars Company's sorters are not putting the correct number of M&M's in each package, mechanical error, etc*…). The **goodness of fit** test we will be using is called a Chi-Square ($\chi 2$) Analysis

Table 3: Small Bag

(Round to the nearest whole number)

Color	Total #	%	Exp	Obs
Brown		0.13		
Blue		0.24		
Orange		0.20		
Green		0.16		
Red		0.13		
Yellow		0.14		

Table 4: Chi-Square Analysis - Small Bag

(Round to the nearest whole number)

Color	Obs	Exp	obs - exp	$(obs - exp)^2$	$\frac{(obs - exp)^2}{exp}$
Brown					
Blue					
Orange					
Green					
Red					
Yellow					
				X^2 Total	
				Degrees of Freedom	

(Round your answers to the nearest hundredth)

Conclusion: ____

____

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Table 5 - Large Bag

(Round to the nearest whole number)

Color	Total #	%	Exp	Obs
Brown		0.13		
Blue		0.24		
Orange		0.20		
Green		0.16		
Red		0.13		
Yellow		0.14		

Table 6: Chi-Square Analysis - Large Bag

(Round to the nearest whole number)

Color	Obs	Exp	obs - exp	$(\text{obs - exp})^2$	$\frac{(\text{obs - exp})^2}{\text{exp}}$
Brown					
Blue					
Orange					
Green					
Red					
Yellow					
				X^2 Total	
				Degrees of Freedom	

(Round your answers to the nearest hundredth)

Conclusion: ____

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Table 7 - BIG Bag

(Round to the nearest whole number)

Color	Total #	%	Exp	Obs
Brown		**0.13**		
Blue		**0.24**		
Orange		**0.20**		
Green		**0.16**		
Red		**0.13**		
Yellow		**0.14**		

Table 8: Chi-Square Analysis - BIG Bag

(Round to the nearest whole number)

Color	Obs	Exp	obs - exp	$(obs - exp)^2$	$\frac{(obs - exp)^2}{exp}$
Brown					
Blue					
Orange					
Green					
Red					
Yellow					
			X^2 Total		
			Degrees of Freedom		

(Round your answers to the nearest hundredth)

Conclusion: ____

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Part IV. Stating Conclusions

Once you have collected your data and analyzed them to get your p-value, you are ready to determine whether your original hypothesis is supported or not. If the p-value in your analysis is 0.05 or less, then the data do not support your null hypothesis with 95% confidence that the observed results would be obtained due to chance alone.

So, as a scientist, you would state your "unacceptable" results in this way:

"The differences observed in the data were statistically significant at the 0.05 level."

You could then add a statement like,

"Therefore, with 95% confidence, the data do not support the hypothesis that..."

This is how a scientist would state "acceptable" results:

"The differences observed in the data were not statistically significant at the 0.05 level."

You could then add a statement like,

"Therefore, with 95% confidence, the data support the hypothesis that..."

And you will see that over and over again in the conclusions of research papers.

Special Note:

The probability decreases as the Chi-Square value increases. Therefore, the lower the Chi- Square value, the higher the probability that the difference between the observed results and the expected results is due to chance alone and you ACCEPT the null hypothesis. Usually, a scientist is hoping to find a low Chi-Square value (ACCEPT H_o) because it means there is a high probability that the deviation from the expected results <u>is due to chance alone</u>. This tells the scientist that the proposed explanation is likely to be correct.
If, however, the Chi-Square value is high (REJECT H_o), it means that there is a low probability that the deviation is due to chance alone. This tells the scientist that the explanation is probably incorrect and that the true reason for the deviation is something other than chance alone. At that point you have some explaining to do and it may be back to the drawing board!

Post-Lab Questions

1. We begin by stating the null hypothesis. Remember, the null hypothesis for a Chi-Square analysis is always the same. What is the null hypothesis for this activity?

Null Hypothesis (Ho): ____

____

____

____

2. What is the number of degrees of freedom? ________
(n - 1)

3. Remember that the Chi-Square value is a measure of the difference between the observed and expected numbers. We are using it to test whether the observed and expected numbers are close enough to accept the null hypothesis *(there is no significant difference AND chance alone can explain the difference*) or so far apart that the null hypothesis must be rejected and there must be more at play here. Based upon the results of this activity, what are your OVERALL conclusions:

Small Bag – Results: **Accept** the null hypothesis **Reject** the null hypothesis

Large Bag – Results: **Accept** the null hypothesis **Reject** the null hypothesis

4. Compare your Chi-square results of the small, large and BIG bag of M&M's?

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5. In order to test the hypothesis that a coin is fair, you toss the coin 100 times and observe that it landed on heads 38 times.

(a) Calculate the observed number of tails and the expected number of heads and tails to fill the information in the table below:

Table 1. Results of Flipping a Coin 100 Times

	Heads	Tails
Observed	38	
Expected		

(b) Use Chi-Square table and formula below to determine if the coin is fair.

CHI-SQUARE TABLE

	Degrees of Freedom							
p	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0.05	3.84	5.99	7.82	9.49	11.07	12.59	14.07	15.51
0.01	6.64	9.32	11.34	13.28	15.09	16.81	18.48	20.09

$$X^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

6. Many casinos use card-dealing machines to deal cards at random. Occasionally the machine is tested to ensure an equal likelihood of dealing for each suit. To conduct the test, 1,500 cards are dealt from the machine while the number of cards in each suit is counted.

(a) Calculate the expected number of cards for each suit and fill that information in the table below:

Table 1.

	Spades	**Diamond**	**Clubs**	**Hearts**
Observed	402	358	273	467
Expected				

(b) Use Chi-Square table and formula below to determine if the discrepancies are significant.

CHI-SQUARE TABLE

	Degrees of Freedom							
p	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0.05	3.84	5.99	7.82	9.49	11.07	12.59	14.07	15.51
0.01	6.64	9.32	11.34	13.28	15.09	16.81	18.48	20.09

$$X^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

* * * BONUS * * *

7. In a certain lizard, eyes can be either black or yellow. When a black-eye lizard is breed with a yellow-eyed lizard, all of the offspring have black eyes. Two black-eyed offspring are crossed, and the result is 72 black eyed lizards, and 28 yellow-eyed lizards.

(a) Using the Punnett Square to the right to help calculate the expected number of black-eye and yellow-eyed offspring and fill that information in the table below:

Table 1.

	Black-eyed	**Yellow-eyed**
Observed	72	28
Expected		

(b) Use Chi-Square table and formula below to calculate the chi-squared value for the null hypothesis that the black eyed parents were heterozygous for eye color.

CHI-SQUARE TABLE

	Degrees of Freedom							
p	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0.05	3.84	5.99	7.82	9.49	11.07	12.59	14.07	15.51
0.01	6.64	9.32	11.34	13.28	15.09	16.81	18.48	20.09

$$X^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$